

Constraint of $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing on warped extra-dimension model

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ABSTRACT: Recent CDF measurement of the $B_s^0 - \bar{B}_s^0$ oscillation frequency at the Tevatron imposes significant constraint on various models for new physics. A warped extra-dimension model with custodial isospin symmetry accommodates the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing at tree level mainly through the Kaluza-Klein gluons. This is due to the misalignment between the bulk gauge eigenstates and the localized Yukawa eigenstates of the bulk fermions. We adopt the universal 5D Yukawa coupling model where all Yukawa couplings are of order one. The SM fermion mass spectra and mixings are controlled by the bulk Dirac mass parameters. With two versions of the hadronic parameter values for $\hat{B}_{B_{d,s}} f_{B_{d,s}}^2$, we investigate the implication of the observed $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings on this model. The CP-violating effects on the B_d system is shown to provide very strong constraint: The first Kaluza-Klein mass of a gluon (M_{KK}) has its lower bound about 3.7 TeV with 1σ uncertainty.

KEYWORDS: B-Physics, Beyond Standard Model.

Contents

1.	Introduction	1
2.	The warped extra-dimension model with custodial symmetry on TeV brane	3
3.	The effects on $B_q^0 - \bar{B}_q^0$ mixing	8
4.	Conclusions	12

1. Introduction

Recently CDF collaboration [1] has measured the oscillation frequency of $B_s^0 - \bar{B}_s^0$ mixing using abundant B_s mesons at the Tevatron.¹ $B_d^0 - \bar{B}_d^0$ mixing had been also measured by BaBar and Belle experiments at the $e^+e^- B$ factories [3]. The current experimental results are [3, 1]

$$\begin{aligned} \Delta M_d^{\text{exp}} &= (0.507 \pm 0.004) \text{ ps}^{-1}, \\ \Delta M_s^{\text{exp}} &= [17.33_{-0.21}^{+0.42}(\text{stat}) \pm 0.07(\text{syst})] \text{ ps}^{-1}. \end{aligned} \tag{1.1}$$

The observed oscillation frequency of $B_q^0 - \bar{B}_q^0$ mixing ($q \in \{d, s\}$) determines the mass difference of $B_q^0 - \bar{B}_q^0$ states, $\Delta M_q \equiv M_{B_q^{\text{heavy}}} - M_{B_q^{\text{light}}}$. For the $B_q^0 - \bar{B}_q^0$ transition amplitude M_{bb}^q defined by $\langle B_q^0 | \mathcal{H}_{\text{eff}}^{\Delta B=2} | \bar{B}_q^0 \rangle = 2M_{B_q} M_{bb}^q$, its absolute value is determined by the mass difference:

$$\Delta M_q = 2 |M_{bb}^q|. \tag{1.2}$$

and the CP-violating phase ϕ_q generates ‘‘mixing-induced’’ CP violation:

$$\phi_q = \arg \left(M_{bb}^q \right). \tag{1.3}$$

As a flavor-changing neutral-current (FCNC) process, $B_q^0 - \bar{B}_q^0$ mixing is a very sensitive probe for the new physics beyond the standard model (SM) since the SM contributions occur only at loop level. If any new physics model make its tree-level contribution to $B_q^0 - \bar{B}_q^0$ mixing, the model would be strongly constrained. In the literature, the constraints on various new models by $B_q^0 - \bar{B}_q^0$ mixing have been extensively discussed [4].

Many new models are theoretically motivated by the gauge hierarchy problem. Among them, a warped extra dimension model by Randall and Sundrum (RS1) [5] has attracted great interest, which solves the gauge hierarchy problem with geometrical suppression of

¹ ΔM_s has been also observed by DØ collaboration [2], as $17 \text{ ps}^{-1} < \Delta M_s < 21 \text{ ps}^{-1}$ at the 90% C.L..

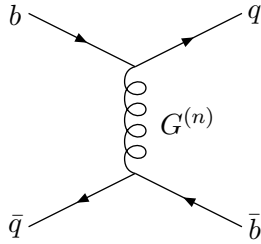


Figure 1: Feynman diagram leading to $B_q^0 - \bar{B}_q^0$ mixing in a warped extra dimension model. $G^{(n)}$ is the n -th KK mode of a gluon.

Planck scale to TeV scale. The RS1 model has one extra spatial dimension of a truncated AdS space, the orbifold of $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$. The fixed point under \mathbb{Z}_2 parity transformation is called the Planck (UV) brane and that under \mathbb{Z}'_2 parity the TeV (IR) brane. In the original RS1 model, the SM fields are localized on the TeV brane in order to avoid any conflict with (most of) experimental data [6, 7]. Later a bulk SM has been widely studied because the phenomenological aspects of the localized field in the 5D theory depend sensitively on the unknown UV physics while those of the bulk field do not [6, 7, 10, 11, 8, 9]. In addition, setting SM fermions in the bulk can explain the enormous mass hierarchy between top quark and neutrino without introducing the hierarchical Yukawa couplings and/or seesaw mechanism [10, 11]. However, many new strongly interacting particles emerge around the TeV scale. The electroweak precision data (EWPD) put very strong constraint mainly due to the lack of SU(2) custodial symmetry in the theory [12–15].

Later the SU(2) custodial symmetry was introduced to be consistent with EWPD. One of the most interesting approaches is the one suggested by Agashe *et al.* [16]. In this model, SU(2) custodial symmetry is induced from AdS₅/CFT feature of bulk gauge symmetry of $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The Higgs boson field remains as an ingredient, which is confined on the TeV brane to avoid another hierarchy problem [7]: Its vacuum expectation value (VEV) generates the SM particle masses. For example, a SM fermion mass is determined by its 5D Yukawa coupling and the overlapping magnitude of the fermion zero mode function on the TeV brane. The Kaluza-Klein (KK) mode function of a bulk fermion is controlled by its bulk Dirac mass parameter c . Unfortunately there is no unique way to determine 5D Yukawa coupling and c 's. One reasonable and attractive choice is to assign universal 5D Yukawa couplings of order one to all fermions [17–20]. Small masses as well as small mixings of the SM fermions (zero modes) are explained by suppressed zero mode functions with moderate values of c 's. Experimental data of SM flavor structure determine the c 's under some reasonable assumptions.

The localized Yukawa couplings generally mix the zero modes of the bulk SM fermions of different generations, while the bulk gauge interactions are flavor-diagonal. This misalignment allows the FCNC coupling at tree level, as depicted in figure 1 [19, 20]. Since the coupling strength of strong interaction is much larger than that of electroweak (EW) interactions, the KK gluon contribution to ΔM_q is dominant. These tree-level contributions to $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings can be *very sensitive probe to the custodial bulk RS1 model*.

This is the primary goal of the paper.

In ref. [20], a general argument on the $\Delta F = 2$ FCNC processes in this model was discussed and the size of the new physics contribution was roughly estimated. With the new experimental results on the $B_s^0 - \bar{B}_s^0$ mixing, more comprehensive and detailed study on this topic is worthwhile. In this paper, we present the full formalism including general complex phases in the left- and right-handed mixing matrices. As shall be shown, the presence of complex phases is crucial especially when we adopt a certain SM calculation of ΔM_q . Even though there is no prior knowledge on the value of the SM phase $\phi_{d(s)}^{\text{SM}}$, the approximate value of ϕ_d^{SM} can be indirectly deduced by combining the value of $|V_{ub}|$ measured from inclusive and/or exclusive tree level $b \rightarrow ul\nu$ decays and the unitary phase angle γ of the SM from the interference between $b \rightarrow c$ and $b \rightarrow u$ transitions to $B \rightarrow DK^{(*)}$. And we can get the constraints on the new physics CP-violating phase ϕ_d^{NP} by comparing the direct measurements of the CP phase from $B \rightarrow J/\psi K_s$ decay [21–23]. We will also examine how sensitive the new physics contribution to $B_q^0 - \bar{B}_q^0$ mixing is to the bulk fermion mass parameter.

The organization of the paper is as follows. In the next section, we briefly review the warped extra dimensional model with $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We also formulate the new physics contribution to $B_q^0 - \bar{B}_q^0$ mixing. In section 3, we summarize the current SM calculations for $B_q^0 - \bar{B}_q^0$, and examine the parameter space where the model is compatible with the observed $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings and the CP phase ϕ_d . We conclude in section 4.

2. The warped extra-dimension model with custodial symmetry on TeV brane

The basic set-up of the model is the same as that of references [19, 20, 17, 24, 18]. We consider $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory in a five-dimensional warped space-time with the metric of

$$ds^2 = e^{-2\sigma(y)}(dt^2 - d\vec{x}^2) - dy^2, \tag{2.1}$$

where y is the fifth dimension coordinate and $\sigma(y) = k|y|$ with k at the Planck scale. The theory is compactified on the $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$ orbifold, which is a circle (with radius r_c) compactified by two reflection symmetries under $\mathbb{Z}_2: y \rightarrow -y$ and $\mathbb{Z}'_2: y' (= y - \pi r_c/2) \rightarrow -y'$. In what follows, we denote \mathbb{Z}_2 parity by P , and \mathbb{Z}'_2 parity by P' . Often conformal coordinate $z \equiv e^{\sigma(y)}/k$ is more useful with the metric of

$$ds^2 = \frac{1}{(kz)^2}(dt^2 - dx^2 - dz^2). \tag{2.2}$$

The orbifold confines the fifth dimension $y \in [0, L(\equiv \pi r_c/2)]$ or $z \in [1/k, 1/(e^{-kL}k)]$. With moderate value of $kL \approx 35$, the IR cut-off $T \equiv e^{-kL}k$ can be at the TeV scale so that the gauge hierarchy problem is solved:

$$T \equiv \epsilon k \sim \text{TeV} \quad \text{with } \epsilon \equiv e^{-kL} \ll 1. \tag{2.3}$$

There are two fixed points in the orbifold of $S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$, the \mathbb{Z}_2 -fixed point at $y = 0$ ($z = 1/k$) called the Planck brane and the \mathbb{Z}'_2 -fixed point at $y = L$ ($z = 1/T$) called the TeV brane.

Among the bulk $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, $SU(2)_R$ symmetry is broken by orbifold boundary conditions on the Planck brane to $U(1)_R$: We impose $(PP') = (-+)$ for $\widetilde{W}_R^{1,2}$. Note that the TeV brane is $SU(2)_R$ symmetric. The $U(1)_R \times U(1)_{B-L}$ is spontaneously broken into $U(1)_Y$ on the Planck brane. Finally the EW symmetry breaking of $SU(2)_L \times U(1)_Y$ is triggered by the VEV of the Higgs field localized on the TeV brane. The SM gauge field is a zero mode of a bulk gauge field with $(++)$ parity.

A five-dimensional gauge field $A^M(x, z)$ (dimension 3/2) is expanded in terms of KK modes:

$$A_\nu(x, z) = \sqrt{k} \sum_n A_\nu^{(n)}(x) f_A^{(n)}(z), \quad (2.4)$$

where the mode function $f_A^{(n)}(z)$ is dimensionless. The zero mode function is

$$f_A^{(0)} = \frac{1}{\sqrt{kL}}, \quad (2.5)$$

and the n -th ($n > 0$) mode function is

$$f_A^{(n)}(z) = \frac{Tz}{N_A^{(n)}} \left[J_1(m_A^{(n)} z) + \beta_A^{(n)} Y_1(m_A^{(n)} z) \right]. \quad (2.6)$$

The following double constraints on $\beta_A^{(n)}$ determines the KK masses for the gauge bosons:

$$\beta_A^{(n)} = -\frac{J_0(m_A^{(n)}/T)}{Y_0(m_A^{(n)}/T)} = -\frac{J_0(m_A^{(n)}/k)}{Y_0(m_A^{(n)}/k)}. \quad (2.7)$$

The normalization constant is

$$N_A^{(n)} = \left| \frac{\left[J_1(x_A^{(n)}) + \beta_A^{(n)} Y_1(x_A^{(n)}) \right]^2}{2} - 2 \left(\frac{\beta_A^{(n)}}{\pi x_A^{(n)}} \right)^2 \right|^{1/2}. \quad (2.8)$$

The five-dimensional action of a bulk fermion $\Psi(x^\mu, y)$ is

$$S_{\text{fermion}} = \int d^4x dy \left[\widehat{\Psi} e^\sigma i \gamma^\mu \partial_\mu \widehat{\Psi} - \frac{1}{2} \widehat{\Psi} \gamma_5 \partial_y \widehat{\Psi} + \frac{1}{2} (\partial_y \widehat{\Psi}) \gamma_5 \widehat{\Psi} + m_D \widehat{\Psi} \widehat{\Psi} \right], \quad (2.9)$$

where $\widehat{\Psi} \equiv e^{-2\sigma} \Psi$ and the 5D Dirac mass is $m_D = ck \text{sign}(y)$. The bulk fermion field is decomposed in terms of KK modes as

$$\widehat{\Psi}(x, z) = \sqrt{k} \sum_n \left[\psi_L^{(n)}(x) f_L^{(n)}(z) + \psi_R^{(n)}(x) f_R^{(n)}(z) \right], \quad (2.10)$$

where $\psi_{R,L}^{(n)}(x) = \frac{1 \pm \gamma^5}{2} \psi^{(n)}$. The Dirac mass parameter c determines the KK mass spectrum and mode functions. Since $\mathbb{Z}_2 \times \mathbb{Z}'_2$ parity of Ψ_L is always opposite to that of Ψ_R , the left-handed SM fermion is the zero mode of a 5D fermion whose left-handed part has $(++)$ parity (and the right-handed part has automatically $(--)$ parity). And the right-handed SM fermion, a singlet under $SU(2)_L$, is the zero mode of *another* 5D fermion whose right-handed part has $(++)$ parity. We have two non-vanishing zero mode functions,

$$f_L^{(0)}(z, c) = \frac{(Tz)^{-c}}{N_L^{(0)}}, \quad f_R^{(0)}(z, c) = \frac{(Tz)^c}{N_R^{(0)}}, \quad (2.11)$$

with the normalization constants of

$$N_L^{(0)} = \epsilon^{-c} \left| \frac{1 - \epsilon^{2c-1}}{2c-1} \right|^{1/2}, \quad N_R^{(0)} = \epsilon^{-c} \left| \frac{1 - \epsilon^{-2c-1}}{-2c-1} \right|^{1/2}. \quad (2.12)$$

Since the $\widetilde{W}_R^{1,2}$ fields with $(-+)$ parity couples two elements of a $SU(2)_R$ doublet, we have an extra $(-+)$ parity fermion in each $SU(2)_R$ doublet. Focusing on the $B_q^0 - \bar{B}_q^0$ mixing which involves only the zero modes for fermions, we consider the fermion fields which contains the $(++)$ parity only. Then the whole SM quarks can be contained in the bulk doublets,

$$Q_i = \begin{pmatrix} u_{iL}^{(++)} \\ d_{iL}^{(++)} \end{pmatrix}, \quad U_i = \begin{pmatrix} u_{iR}^{(++)} \\ D_{iR}^{(-+)} \end{pmatrix}, \quad D_i = \begin{pmatrix} U_{iR}^{(-+)} \\ d_{iR}^{(++)} \end{pmatrix}, \quad (2.13)$$

where i is the generation index. Note that nine Dirac mass parameters (c_{Q_i} , c_{U_i} , and c_{D_i}) determine the zero-mode functions and KK mass spectra in the quark sector.

To generate the SM fermion masses, we use the localized Higgs field on the TeV brane as the usual SM Higgs mechanism. The Yukawa interaction between bulk quarks and Higgs is [30]

$$S_Y = - \int d^4x dy \frac{\delta(y-L)}{T} \left[\lambda_{5ij}^u \bar{u}_{iR}(x, y) \tilde{H}^\dagger(x) \hat{Q}_{jL}(x, y) + \lambda_{5ij}^d \bar{d}_{iR}(x, y) H^\dagger \hat{Q}_{jL} + h.c. \right], \quad (2.14)$$

where $H = e^{-kL} \phi(x)$ is canonically normalized Higgs field, $\tilde{H} = i\tau_2 H^*$, and i, j are the generation indices. The boundary mass term is realized when the Higgs field develops the VEV of $\langle H \rangle = v$. The SM mass matrices for up- and down-type quarks are then, for $q = U, D$,

$$M_{ij}^q = v \lambda_{5ij}^q \frac{k}{T} f_R^{(0)}(z, c_{q_i}) f_L^{(0)}(z, c_{q_j}) \Big|_{z=1/T}. \quad (2.15)$$

The mass matrix $M^{u,d}$ is diagonalized by bi-unitary transformation:

$$U_{qR} M^q U_{qL}^\dagger = M_{(\text{diag})}^q \quad \text{for } q = u, d. \quad (2.16)$$

The mass eigenstates are

$$\chi_{qL} = U_{qL} \psi_{qL}^{(0)}, \quad \chi_{qR} = U_{qR} \psi_{qR}^{(0)}. \quad (2.17)$$

The Cabibbo-Kobayashi-Maskawa (CKM) matrix [25] is defined as $V^{\text{CKM}} = U_{\text{uL}}^\dagger U_{\text{dL}}$. A natural choice for U_{uL} and U_{dL} is that both mixing matrices have similar form of the CKM matrix. This choice of mixing is reasonable since the u_L and d_L , which belong to the same $SU(2)_L$ doublet, have the same bulk mass. We parameterize

$$(U_{\text{qL}})_{ij} = \kappa_{ij} V_{ij}^{\text{CKM}}, \quad (2.18)$$

where κ_{ij} 's are complex parameters of order one. To avoid order changing during the diagonalization of matrix, κ^2 should be greater than $\sin \theta_c$ (θ_c is the Cabibbo angle) and smaller than $1/\sin \theta_c$. Therefore, we assume $|\kappa_{ij}| \in [1/\sqrt{2}, 2]$.

In the SM, the huge mass difference between electron and top quark is explained by hierarchical Yukawa couplings. Even though the SM fermion mass in this model is also generated through the VEV of the localized Higgs field, the mass hierarchy can be attributed to different overlapping probability (by controlling c 's) of the zero-mode function on the TeV brane. The SM fermion mass spectra have been studied for various values of the Dirac mass parameter c 's, and found that the large SM fermion mass hierarchy can be explained without introducing large hierarchy in the model parameters [19, 20, 17, 24, 18].

The structure of Yukawa couplings is arbitrary in this model. One popular choice is to assume that all of the 5D Yukawa couplings (to all flavors) have almost universal strength of order one: The fermion hierarchy is generated only by different bulk Dirac mass parameters. Since the mass eigenvalues and CKM parameters are empirically fixed, only unknowns are λ_5^q of eq. (2.15) and κ_{ij} in eq. (2.18). Since both parameters are all assumed to be of order one, the numerical ambiguity of λ_5^q can be absorbed into κ by the redefinition of parameters.

In this set-up (where the 5D Yukawa couplings are universal and the mixing matrices are CKM-like), the SM quark mass spectrum is reproduced with the following Dirac mass parameters [18]:

$$\begin{aligned} c_{Q1} &\simeq 0.61, & c_{Q2} &\simeq 0.56, & c_{Q3} &\simeq 0.3_{-0.04}^{+0.02}, \\ c_{D1} &\simeq -0.66, & c_{D2} &\simeq -0.61, & c_{D3} &\simeq -0.56, \\ c_{U1} &\simeq -0.71, & c_{U2} &\simeq -0.53, & 0 &\lesssim c_{U3} \lesssim 0.2. \end{aligned} \quad (2.19)$$

As discussed before, the hierarchical SM mass spectrum can be explained with moderate values of the model parameters c 's. At first glance, rather definite values of $c_{Q_{1,2}}$ seem unnatural, compared with $c_{Q_3} \in [0.26, 0.32]$. This is due to the behavior of the zero mode function, defined in eq. (2.11), in the fermion mass matrix of eq. (2.15). For $c > 0.5$, the $f_L^{(0)}$ is very sensitive to the change of c , leading to strong constraint on c from the fermion mass hierarchy: The values of $c_{Q_{1,2}}$ are practically determined by the SM quark mass hierarchy. For $c < 0.5$, however, the zero mode function does not change that much. The value of c_{Q_3} has some range, though small if we consider EW precision data and Yukawa coupling around $[1/\sqrt{2}, 2]$ [16].

The 5D action for the gauge interaction of a fermion is

$$S_G = \int d^4x dy \sqrt{G} \frac{g_5}{\sqrt{k}} \bar{\Psi}(x, y) i\gamma^\mu A_\mu(x, y) \Psi(x, y), \quad (2.20)$$

where g_5 is the 5D dimensionless gauge coupling, and $\Psi = Q_{iL}, u_{iR}, d_{iR}$. For the $B_q^0 - \bar{B}_q^0$ mixing, only the zero modes of Q_{iL} and d_{iR} are relevant. With the preferred values of c_i 's in eq. (2.19), the zero modes of Q_{iL} are dominant over those of d_{iR} . Therefore, we take the contributions of the zero modes of Q_{iL} only. Substituting eqs. (2.5), (2.6) and (2.11) into eq. (2.20) leads to the four-dimensional gauge couplings, defined by

$$\mathcal{L}_{4D} = g_{\text{SM}} \bar{\psi}_{jL}^{(0)} i\gamma^\mu \psi_{jL}^{(0)} A_\mu(0) + g_{\text{SM}} \sum_{n=1}^{\infty} \hat{g}_j^{(n)}(c_{Q_i}) \bar{\psi}_{jL}^{(0)} i\gamma^\mu \psi_{jL}^{(0)} A_\mu^{(n)}, \quad (2.21)$$

where $g_{\text{SM}} = g_5/\sqrt{kL}$ and

$$\hat{g}_i^{(n)}(c_{Q_i}) = \sqrt{kL} \int dzk \left[f_L^{(0)}(z, c_{Q_i}) \right]^2 f_A^{(n)}(z). \quad (2.22)$$

With the preferred Dirac mass parameters in eq. (2.19), we have the following hierarchy in $\hat{g}_i^{(n)}$'s:

$$\hat{g}_1^{(n)} \sim \lambda^2 \hat{g}_3^{(n)}, \quad \hat{g}_2^{(n)} \sim \lambda^2 \hat{g}_3^{(n)}, \quad (2.23)$$

where $\lambda = \sin \theta_c \simeq 0.22$. Thus, $\hat{g}_3^{(n)}$ is dominant, of which the value is determined by c_{Q_3} :

$$\hat{g}_3^{(1)} = \begin{cases} 1.97 & \text{for } c_{Q_3} = 0.3 ; \\ 1.80 & \text{for } c_{Q_3} = 0.3 + 0.02 ; \\ 2.31 & \text{for } c_{Q_3} = 0.3 - 0.04 . \end{cases} \quad (2.24)$$

The localized Yukawa interaction causes the mixing between the gauge eigenstates and the mass eigenstates as in eq. (2.17). The n -th KK gauge interaction among down-type quark mass eigenstates is

$$\mathcal{L}_{4D} \supset g_{\text{SM}} \sum_{i,j=1}^3 \sum_{n=1}^{\infty} K_{ij}^{(n)} \bar{d}_{iL}^{(0)} i\gamma^\mu d_{jL}^{(0)} A_\mu^{(n)}, \quad (2.25)$$

where d_{iL} is the mass eigenstate, and

$$K_{ij}^{(n)} = \sum_{k=1}^3 (U_{dL})_{ik} \hat{g}_k^{(n)}(c_{Q_k}) (U_{dL}^\dagger)_{kj}. \quad (2.26)$$

Note that if all of the c_{Q_i} are the same for three generations so that $\hat{g}_k^{(n)}(c_{Q_k})$ is common, the $K_{ij}^{(n)}$ is proportional to δ_{ij} : No generation mixing and thus no contribution to $B_q^0 - \bar{B}_q^0$ mixing occur.

Through the n -th KK mode of a gluon exchange, $B_q^0 - \bar{B}_q^0$ mixing amplitude is proportional to

$$M_{bb}^{q(n)} \propto \left(K_{q3}^{(n)} \right)^2 \simeq \left[\kappa_{33} \kappa_{q3} V_{tb}^* V_{tq} \hat{g}_3^{(n)}(c_{Q_3}) \right]^2, \quad (2.27)$$

where for the second equality we have included only the dominant $\hat{g}_3^{(n)}$ as in eq. (2.23).

3. The effects on $B_q^0 - \bar{B}_q^0$ mixing

In the bulk RS model, the $B_q^0 - \bar{B}_q^0$ mixing is due to the SM box diagrams and the RS KK gluons:

$$M_{bb}^q = M_{bb}^{q,\text{SM}} \left(1 + \frac{M_{bb}^{q,\text{RS}}}{M_{bb}^{q,\text{SM}}} \right). \quad (3.1)$$

We parameterize, following the notation in ref. [23], the new physics effect by

$$r_q e^{i\sigma_q} = \frac{M_{bb}^{q,\text{RS}}}{M_{bb}^{q,\text{SM}}}, \quad (3.2)$$

where $r_q \geq 0$ and σ_q is real. The r_q and σ_q are constrained by the experimental result for ΔM_q and the theoretical calculation of the ΔM_q^{SM} , of which the ratio is defined by ρ_q :

$$\rho_q \equiv \left| \frac{\Delta M_q}{\Delta M_q^{\text{SM}}} \right| = \left| \frac{M_{bb}^{q,\text{SM}} + M_{bb}^{q,\text{RS}}}{M_{bb}^{q,\text{SM}}} \right| = \sqrt{1 + 2r_q \cos \sigma_q + r_q^2}. \quad (3.3)$$

The CP-violating phase ϕ_d can be divided into the phase from the SM and that from New Physics (NP) contributions.

$$\phi_q = \phi_q^{\text{SM}} + \phi_q^{\text{NP}} = \phi_q^{\text{SM}} + \arg(1 + r_q e^{i\sigma_q}). \quad (3.4)$$

The NP phase ϕ_q^{NP} is determined by r_q and σ_q . Therefore, the observation of ϕ_q^{NP} can constrain r_q and σ_q , independently of ρ_q , through the following relations:

$$\sin \phi_q^{\text{NP}} = \frac{r_q \sin \sigma_q}{\sqrt{1 + 2r_q \cos \sigma_q + r_q^2}}, \quad (3.5)$$

$$\cos \phi_q^{\text{NP}} = \frac{1 + r_q \cos \sigma_q}{\sqrt{1 + 2r_q \cos \sigma_q + r_q^2}}. \quad (3.6)$$

The SM contribution is poorly known mainly due to non-perturbative nature of the input hadronic parameters even though experiments have measured ΔM_d and ΔM_s with high precision. M_{12}^q within the SM is

$$M_{bb}^{q,\text{SM}} = \frac{G_F^2 m_W^2}{12\pi^2} M_{B_q} \hat{\eta}^B \hat{B}_{B_q} f_{B_q}^2 (V_{td}^* V_{tb})^2 S_0(x_t), \quad (3.7)$$

where $x_t = m_{\text{top}}^2/m_W^2$, and S_0 is an ‘‘Inami-Lim’’ function [26]. For CKM parameters, we used $|V_{td}^* V_{tb}| = (8.6 \pm 1.3) \times 10^{-3}$ and $|V_{ts}^* V_{tb}| = (41.3 \pm 0.7) \times 10^{-3}$ [22, 23]. Common quantities for both B_d and B_s system are m_W and a short-distance QCD correction $\hat{\eta}^B = 0.552$ [27]. Flavor dependent and non-perturbative quantities are the bag parameter \hat{B}_{B_q} and the decay constant f_{B_q} . $r_q e^{i\sigma_q}$ in this model becomes

$$r_q e^{i\sigma_q} \equiv \frac{M_{bb}^{q,\text{RS}}}{M_{bb}^{q,\text{SM}}} = \frac{16\pi^2}{N_C} \frac{8g_s^2}{g^4 S_0(x_t)} m_W^2 \kappa_{33}^2 \kappa_{q3}^2 \sum_{n=1} \left(\frac{\hat{g}_3^{(n)}(c_{Q_3})}{m_A^{(n)}} \right)^2, \quad (3.8)$$

where $\sigma_q = 2 \arg(\kappa_{3q})$. r_q represents the magnitude of the new physics effect, and σ_q is a new source of CP violation. Note that the parametrization in eq. (2.18) removes the CKM factor in the ratio.

There are several estimates of the SM values for ΔM_q^{SM} . We use the following two results for the input hadronic parameters $\hat{B}_{B_{d,s}} f_{B_{d,s}}^2$. The first one is from the most recent (unquenched) simulation by JLQCD collaboration [28], with non-relativistic b quark and two flavors of dynamical light quarks. The second one is from combined results, denoted by (HP+JL)QCD: Lacking any direct calculation of \hat{B}_{B_q} with three dynamical flavors, it has been suggested to combine the results of f_{B_q} from HPQCD collaboration [29] with that of \hat{B}_{B_q} from JLQCD. Two numerical results are

$$\Delta M_d^{\text{SM}} = \begin{cases} 0.52 \pm 0.17_{+0.13}^{-0.09} \text{ ps}^{-1}, & \text{for JLQCD;} \\ 0.69 \pm 0.13 \pm 0.08 \text{ ps}^{-1}, & \text{for (HP+JL)QCD,} \end{cases} \quad (3.9)$$

$$\Delta M_s^{\text{SM}} = \begin{cases} 16.1 \pm 2.8 \text{ ps}^{-1}, & \text{for JLQCD;} \\ 23.4 \pm 3.8 \text{ ps}^{-1}, & \text{for (HP+JL)QCD,} \end{cases} \quad (3.10)$$

where the first error in eq. (3.9) is from the uncertainties of the CKM angle γ and R_b , while the second one is from those of $f_{B_d} \hat{B}_{B_d}^{1/2}$.

From the relation of $\Delta M_q^{\text{RS}} = \Delta M_q^{\text{SM}} \rho_q$ in this model, we compute ρ_q by using the experimental values and the SM values for $\Delta M_{d,s}$ as

$$\rho_d = \begin{cases} 0.97 \pm 0.33_{+0.26}^{-0.17}, & \text{for JLQCD;} \\ 0.75 \pm 0.25 \pm 0.16, & \text{for (HP+JL)QCD,} \end{cases} \quad (3.11)$$

$$\rho_s = \begin{cases} 1.08_{-0.01}^{+0.03}(\text{exp}) \pm 0.19(\text{th}), & \text{for JLQCD;} \\ 0.74_{-0.01}^{+0.02}(\text{exp}) \pm 0.18(\text{th}), & \text{for (HP+JL)QCD.} \end{cases}$$

The CP-violating phase ϕ_d associated with $B_d^0 - \bar{B}_d^0$ is well measured, of which the most recent average is [3]

$$(\sin \phi_d)_{cc\bar{s}} = \sin(2\beta + \phi_d^{\text{NP}}) = 0.687 \pm 0.032, \quad (3.12)$$

where we have used $\phi_d^{\text{SM}} = 2\beta$. The angle β depends on two tree-level quantities of R_b and γ . In spite of very large uncertainty in the angle γ , the angle β can be reasonably well constrained since β is very weakly affected by γ . The new CP-violating phase is shown to be

$$\phi_d^{\text{NP}}|_{\text{incl}} = -(10.1 \pm 4.6)^\circ, \quad \phi_d^{\text{NP}}|_{\text{excl}} = -(2.5 \pm 8.0)^\circ. \quad (3.13)$$

The detailed explanation for $\phi_d^{\text{NP}}|_{\text{incl}}$ and $\phi_d^{\text{NP}}|_{\text{excl}}$ is referred to ref. [23]. Note that $\sin \sigma_d$ has the same sign with $\sin \phi_d^{\text{NP}}$ through the relation in eq. (3.5) with $r_q \geq 0$: For $\phi_d^{\text{NP}}|_{\text{incl}}$, therefore, we explore the parameter σ_d in $[\pi, 2\pi]$ to satisfy eq. (3.6).

The new contribution depends on two parameters, σ_d and M_{KK} . Here M_{KK} denotes the TeV scale mass of the first KK mode of a gluon. In figure 2, we show in the parameter space of $(\sigma_d, M_{\text{KK}})$ the contours for ρ_d (three red lines) and $\phi_d^{\text{NP}}|_{\text{incl}}$ (three blue lines) with 1σ hadronic uncertainty. With $c_{Q_3} = 0.3$ and $\kappa = 1$, we use the JLQCD ones for the SM

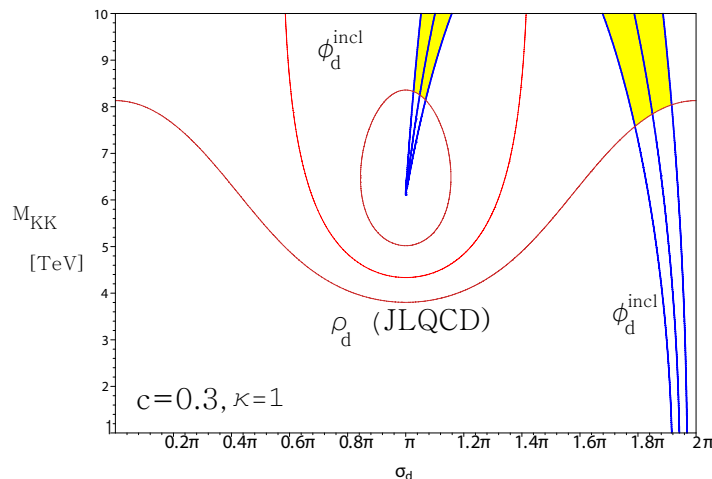


Figure 2: Allowed parameter space of (σ_d, M_{KK}) by ρ_d and $\phi_d^{NP}|_{incl}$. Three red (thin) lines satisfy the observed ρ_d and three blue (thick) lines for $\phi_d^{NP}|_{incl}$, both with 1σ uncertainty. We set $c_{Q_3} = 0.3$ and $\kappa = 1$, and use JLQCD hadronic input parameters.

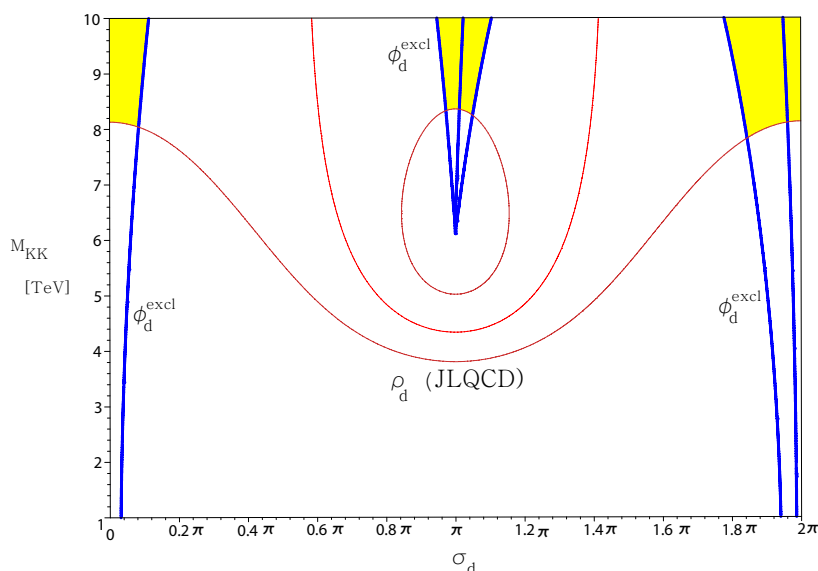


Figure 3: The same plots as in figure 2 except for $\phi_d^{NP}|_{excl}$.

results. The yellow region is allowed by two observation of ρ_d and $\phi_d^{NP}|_{incl}$. Only with the ρ_d constraint, M_{KK} can be as low as about 4 TeV and σ_d in the whole range can be accepted. The ϕ_d^{NP} constraint is quite strong to raise the allowed M_{KK} above 8 TeV. This heavy KK mode is practically impossible to probe at LHC. In addition, the allowed region for σ_d is quite limited to two small regions around π and 1.8π .

Figure 3 shows the same plots but with different CP-violating phase $\phi_d^{NP}|_{excl}$. Large uncertainty in $\phi_d^{NP}|_{excl}$ does not lower the allowed M_{KK} significantly. In figure 4, we

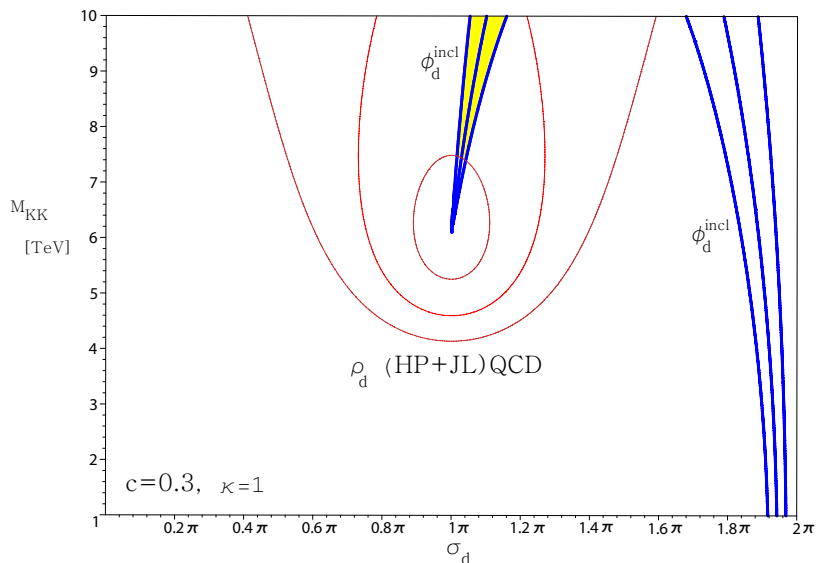


Figure 4: The same plots as in figure 2 for the (HP+JL)QCD hadronic input parameters.

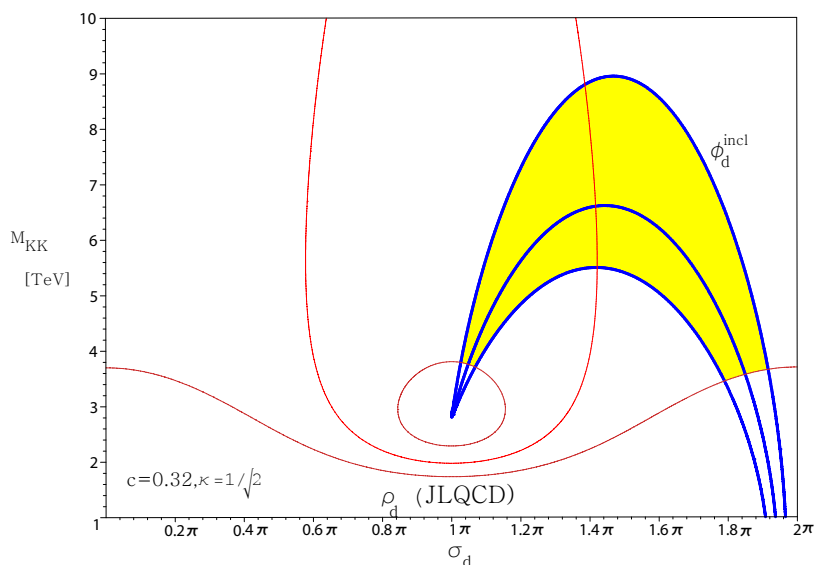


Figure 5: The same plots as in figure 2 but with $c_{Q_3} = 0.32$ and $\kappa = 1/\sqrt{2}$.

present the same allowed parameter space but with different hadronic input parameters, the (HP+JL)QCD one. The ρ_d constraint becomes stronger, which narrows the allowed σ_d region: σ_d is limited around $[\pi, 1.2\pi]$. The lowest allowed M_{KK} is a little reduced, but not significantly. It is still difficult to produce at LHC.

In figure 5, we consider other values for c_{q_3} and κ to see how much they affect the lowest allowed value for M_{KK} . As can be seen from eq. (3.8), the smaller κ and $\hat{g}_3^{(1)}$ allow the

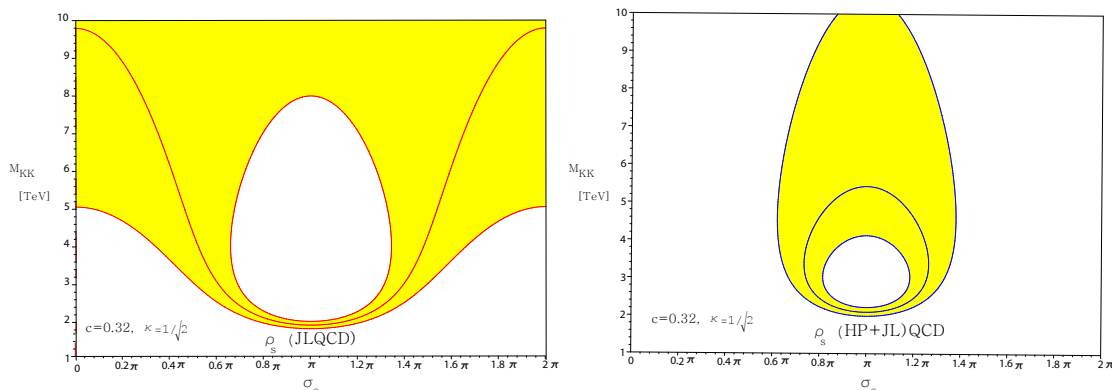


Figure 6: The allowed parameter space of $(\sigma_s, M_{\text{KK}})$ from the observed ΔM_s . We set $c_{Q_3} = 0.32$ and $\kappa = 1/\sqrt{2}$ and used JLQCD and (HP+JL)QCD hadronic input parameters.

lower M_{KK} value. In figure 5, we present the allowed parameter space with $c_{Q_3} = 0.32$ and $\kappa = 1/\sqrt{2}$. The allowed region of σ_d is significantly extended into $[\pi, 1.9\pi]$. The allowed M_{KK} is also remarkably reduced around 3.7 TeV so that the first KK gauge boson can be marginally produced at LHC. Note that we can choose even lower $\kappa < 1/\sqrt{2}$ to allow lower M_{KK} . However, such a choice may ruin the consistency of the bulk mass form in eq. (2.19).

The CP-violating phase ϕ_s , which enters the $B_s^0 - \bar{B}_s^0$ mixing-induced CP-violation, has not been constrained to this day. The σ_s measurement in the future experiment is of great significance. For example, the $A_{\text{CP}}(B_x \rightarrow \psi\phi)$ and the semi-leptonic asymmetry A_{SL}^s have very suppressed contribution from the SM. Any significant measurement can indicate the new physics effect, which the bulk Randall-Sundrum model under consideration can provide.

In figure (6), we plot the allowed region by the observed ΔM_s . We consider the $c_{Q_3} = 0.32$ and $\kappa = 1/\sqrt{2}$ case with both of JLQCD and (HP+JL)QCD hadronic input parameters. Without any information on ϕ_s , there exist narrow but substantial region for $M_{\text{KK}} \simeq 2$ TeV. However, this region is excluded by the observed ϕ_d .

4. Conclusions

The warped extra dimensional model with custodial isospin symmetry can contribute to $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing at tree level, dominantly through the Kaluza-Klein modes of gluons. This FCNC process at tree level originates from the mixing between the gauge eigenstates and the mass eigenstates due to the flavor-mixing Yukawa couplings localized on the TeV brane: Even though the FCNC among the SM fermions (or zero modes of the bulk fermion) is absent in the 5D gauge interaction, the localized Yukawa couplings can mix the gauge eigenstates. We assume the simplest set-up for the SM mass spectra such that all the 5D Yukawa couplings are of the same order and the mixing matrices have the same form as the CKM matrix. This assumption almost fixes the bulk Dirac mass parameters c_i for each 5D fermion. With the suggested c_i , we have calculated the new contributions to the

$B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings, and compare with the recent experimental results. Main uncertainties are from the hadronic input parameters. However, we found that the lower bound on M_{KK} by the observed $B_q^0 - \bar{B}_q^0$ mixing is, irrespective to hadronic uncertainties, rather high above ~ 3.7 TeV. The LHC can marginally produce the KK gluons. The strongest constraint comes from the observation of the CP-violating phase ϕ_d .

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References

- [1] CDF coll., G. Gomez-Ceballos, talk at FPCP 2006.
- [2] D0 collaboration, V.M. Abazov et al., *First direct two-sided bound on the B/s_0 oscillation frequency*, *Phys. Rev. Lett.* **97** (2006) 021802 [[hep-ex/0603029](#)].
- [3] HEAVY FLAVOR AVERAGING GROUP (HFAG) collaboration, E. Barberio et al., *Averages of B -hadron properties at the end of 2005*, [hep-ex/0603003](#).
- [4] M. Carena, A. Menon, R. Noriega-Papaqui, A. Szyrkman and C.E.M. Wagner, *Constraints on B and Higgs physics in minimal low energy supersymmetric models*, *Phys. Rev.* **D 74** (2006) 015009 [[hep-ph/0603106](#)];
M. Ciuchini and L. Silvestrini, *Upper bounds on SUSY contributions to $b \rightarrow s$ transitions from B/s -anti- B/s mixing*, *Phys. Rev. Lett.* **97** (2006) 021803 [[hep-ph/0603114](#)];
L. Velasco-Sevilla, *Impact of $\Delta(m(B/s))$ on the determination of the unitary triangle and bounds on physics beyond the standard model*, [hep-ph/0603115](#);
M. Endo and S. Mishima, *Constraint on right-handed squark mixings from $B/s - \text{anti} - B/s$ mass difference*, *Phys. Lett.* **B 640** (2006) 205 [[hep-ph/0603251](#)];
Z. Ligeti, M. Papucci and G. Perez, *Implications of the measurement of the B/s_0 -anti- B/s_0 mass difference*, [hep-ph/0604112](#);
M. Blanke, A.J. Buras, D. Guadagnoli and C. Tarantino, *Minimal flavour violation waiting for precise measurements of $\Delta(m(s))$, $|V(ub)|$, γ and $B/s, d_0 \rightarrow \mu^+ \mu^-$* , *JHEP* **10** (2006) 003 [[hep-ph/0604057](#)];
J. Foster, K.-i. Okumura and L. Roszkowski, *New constraints on SUSY flavour mixing in light of recent measurements at the Tevatron*, *Phys. Lett.* **B 641** (2006) 452 [[hep-ph/0604121](#)];
Y. Grossman, Y. Nir and G. Raz, *Constraining the phase of $b/s - \text{anti-}b/s$ mixing*, *Phys. Rev. Lett.* **97** (2006) 151801 [[hep-ph/0605028](#)];
K. Cheung, C.-W. Chiang, N.G. Deshpande and J. Jiang, *Constraints on flavor-changing Z' models by B/s mixing, Z' production and $B/s \rightarrow \mu^+ \mu^-$* , [hep-ph/0604223](#);
G. Isidori and P. Paradisi, *Hints of large $\tan \beta$ in flavour physics*, *Phys. Lett.* **B 639** (2006) 499 [[hep-ph/0605012](#)];
S. Khalil, *Supersymmetric contribution to the CP asymmetry of $B \rightarrow J/\psi \Phi$ in the light of recent B/s -anti- B/s measurements*, [hep-ph/0605021](#);
A. Datta, *B/s mixing and new physics in hadronic $B \rightarrow s$ anti- q q transitions*, [hep-ph/0605039](#);

- S. Baek, *B/s-anti-b/s mixing in the MSSM scenario with large flavor mixing in the LL/RR sector*, *JHEP* **09** (2006) 077 [[hep-ph/0605182](#)];
- X.-G. He and G. Valencia, *anti-B/s B/s mixing constraints on FCNC and a non-universal Z'*, *Phys. Rev. D* **74** (2006) 013011 [[hep-ph/0605202](#)];
- R. Arnowitt, B. Dutta, B. Hu and S. Oh, *B/s-anti-B/s mixing and its implication for $B \rightarrow s$ transitions in supersymmetry*, *Phys. Lett. B* **641** (2006) 305 [[hep-ph/0606130](#)];
- S. Baek, J.H. Jeon and C.S. Kim, *B/s0-anti-B/s0 mixing in leptophobic Z' model*, *Phys. Lett. B* **641** (2006) 183 [[hep-ph/0607113](#)].
- [5] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [[hep-ph/9905221](#)].
- [6] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Bulk gauge fields in the Randall-Sundrum model*, *Phys. Lett. B* **473** (2000) 43 [[hep-ph/9911262](#)].
- [7] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, *Bulk standard model in the Randall-Sundrum background*, *Phys. Rev. D* **62** (2000) 084025 [[hep-ph/9912498](#)].
- [8] S.J. Huber and Q. Shafi, *Higgs mechanism and bulk gauge boson masses in the randall-sundrum model*, *Phys. Rev. D* **63** (2001) 045010 [[hep-ph/0005286](#)].
- [9] S.J. Huber and Q. Shafi, *Fermion masses, mixings and proton decay in a Randall-Sundrum model*, *Phys. Lett. B* **498** (2001) 256 [[hep-ph/0010195](#)].
- [10] Y. Grossman and M. Neubert, *Neutrino masses and mixings in non-factorizable geometry*, *Phys. Lett. B* **474** (2000) 361 [[hep-ph/9912408](#)].
- [11] T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys. B* **586** (2000) 141 [[hep-ph/0003129](#)].
- [12] C.S. Kim, J.D. Kim and J.-h. Song, *Top quark Kaluza-Klein mode mixing in the randall-sundrum bulk standard model and $b \rightarrow x/s$ gamma*, *Phys. Rev. D* **67** (2003) 015001 [[hep-ph/0204002](#)].
- [13] C. Csaki, J. Erlich and J. Terning, *The effective Lagrangian in the Randall-Sundrum model and electroweak physics*, *Phys. Rev. D* **66** (2002) 064021 [[hep-ph/0203034](#)].
- [14] J.L. Hewett, F.J. Petriello and T.G. Rizzo, *Precision measurements and fermion geography in the Randall-Sundrum model revisited*, *JHEP* **09** (2002) 030 [[hep-ph/0203091](#)].
- [15] G. Burdman, *Constraints on the bulk standard model in the Randall-Sundrum scenario*, *Phys. Rev. D* **66** (2002) 076003 [[hep-ph/0205329](#)].
- [16] K. Agashe, A. Delgado, M.J. May and R. Sundrum, *RS1, custodial isospin and precision tests*, *JHEP* **08** (2003) 050 [[hep-ph/0308036](#)].
- [17] S.J. Huber, *Flavor violation and warped geometry*, *Nucl. Phys. B* **666** (2003) 269 [[hep-ph/0303183](#)].
- [18] S. Chang, C.S. Kim and M. Yamaguchi, *Hierarchical mass structure of fermions in warped extra dimension*, *Phys. Rev. D* **73** (2006) 033002 [[hep-ph/0511099](#)].
- [19] K. Agashe, G. Perez and A. Soni, *B-factory signals for a warped extra dimension*, *Phys. Rev. Lett.* **93** (2004) 201804 [[hep-ph/0406101](#)].
- [20] K. Agashe, G. Perez and A. Soni, *Flavor structure of warped extra dimension models*, *Phys. Rev. D* **71** (2005) 016002 [[hep-ph/0408134](#)].

- [21] K. Agashe, M. Papucci, G. Perez and D. Pirjol, *Next to minimal flavor violation*, [hep-ph/0509117](#).
- [22] UTFIT collaboration, M. Bona et al., *The UTFit collaboration report on the unitarity triangle beyond the standard model: spring 2006*, *Phys. Rev. Lett.* **97** (2006) 151803 [[hep-ph/0605213](#)];
UTFIT collaboration, M. Bona et al., *The UTFit collaboration report on the unitarity triangle beyond the standard model: spring 2006*, *Phys. Rev. Lett.* **97** (2006) 151803 [[hep-ph/0605213](#)].
- [23] P. Ball and R. Fleischer, *Probing new physics through B mixing: status, benchmarks and prospects*, *Eur. Phys. J. C* **48** (2006) 413 [[hep-ph/0604249](#)].
- [24] G. Moreau and J.I. Silva-Marcos, *Flavour physics of the RS model with KK masses reachable at LHC*, *JHEP* **03** (2006) 090 [[hep-ph/0602155](#)].
- [25] N. Cabibbo, *Unitary symmetry and leptonic decays*, *Phys. Rev. Lett.* **10** (1963) 531;
M. Kobayashi and T. Maskawa, *CP violation in the renormalizable theory of weak interaction*, *Prog. Theor. Phys.* **49** (1973) 652.
- [26] T. Inami and C.S. Lim, *Effects of superheavy quarks and leptons in low-energy weak processes $k(l) \rightarrow \mu\bar{\mu}$, $k+ \rightarrow \pi^+$ neutrino anti-neutrino and $k0 \leftarrow \bar{k}0$* , *Prog. Theor. Phys.* **65** (1981) 297.
- [27] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)].
- [28] JLQCD collaboration, S. Aoki et al., *$B0$ anti- $B0$ mixing in unquenched lattice QCD*, *Phys. Rev. Lett.* **91** (2003) 212001 [[hep-ph/0307039](#)].
- [29] HPQCD collaboration, A. Gray et al., *The B meson decay constant from unquenched lattice QCD*, *Phys. Rev. Lett.* **95** (2005) 212001 [[hep-lat/0507015](#)].
- [30] S. Chang, S.C. Park and J. Song, *Kaluza-Klein masses of bulk fields with general boundary conditions in AdS_5* , *Phys. Rev. D* **71** (2005) 106004 [[hep-ph/0502029](#)].